Find the limit or indicate that it does not exist.

$$\lim_{t\to\pi}\left[\left(\sin\frac{5}{3}t\right)i + \left(\cos\frac{7}{6}t\right)j + \left(\tan\frac{5}{6}t\right)k\right]$$

Trigonometry Ratio Table

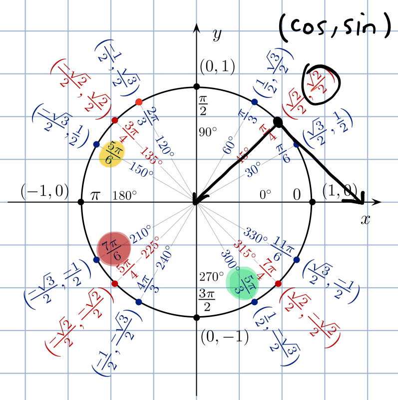
Angles (In Degrees)	$0°$	$30°$	$45°$	$60°$	$90°$	$180°$	$270°$	$360°$
Angles (In Radians)	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π
sin	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	0	-1	0
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	-1	0	1
tan	0	$\frac{1}{\sqrt{3}}$	1	$\sqrt{3}$	Not Defined	0	Not Defined	1
cot	Not Defined	$\sqrt{3}$	1	$\frac{1}{\sqrt{3}}$	0	Not Defined	0	Not Defined
csc	Not Defined	2	$\sqrt{2}$	$\frac{2}{\sqrt{3}}$	1	Not Defined	-1	Not Defined
sec	1	$\frac{2}{\sqrt{3}}$	$\sqrt{2}$	2	Not Defined	-1	Not Defined	1

Find the limit of each component scalar function

$$\lim_{t\to\pi}\left(\sin\frac{5}{3}t\right) = -\frac{\sqrt{3}}{2}$$

$$\lim_{t\to\pi}\left(\cos\frac{7}{6}t\right) = -\frac{\sqrt{3}}{2}$$

$$\lim_{t\to\pi}\left(\tan\frac{5}{6}t\right) = \frac{\frac{1}{2}}{-\frac{\sqrt{3}}{2}} = \frac{1}{2}\cdot -\frac{2}{\sqrt{3}} = \frac{-2}{2\sqrt{3}} = \frac{-1}{\sqrt{3}}$$



thus, the limit is:

$$\lim_{t\to\pi}\left[\left(\sin\frac{5}{3}t\right)i + \left(\cos\frac{7}{6}t\right)j + \left(\tan\frac{5}{6}t\right)k\right] = -\frac{\sqrt{3}}{2}i - \frac{\sqrt{3}}{2}j - \frac{1}{\sqrt{3}}k$$

The position of a particle in the xy-plane at time t is $r(t) = (t-5)i + (t^2-4)j$.

Find an equation in x and y whose graph is the path of the particle. Then find the particle's velocity and acceleration vectors at $t=1$.

given that $r(t) = (t-5)i + (t^2-4)j$

then $r(t) = xi + yj$.

$x = t - 5 \rightarrow t = x + 5$

$y = t^2 - 4$

eliminate t from x and y.

$y = (x+5)^2 - 4$

$y = x^2 + 10x + 5^2 - 4 \quad (x+a)^2 = x^2 + 2ax + a^2$

$y = x^2 + 10x + 21$

Now, velocity of the particle

position vector $r(t)$

$$V(t) = \frac{dr}{dt} = \frac{df}{dt}i = \frac{dg}{dt}j$$

Differentiate x-component $f(t) = t - 5$

$$\frac{df}{dt} = t - 5 \quad d/dx\,(x) = 1$$

$$\frac{df}{dt} = 1$$

Differentiate y-component $f(t) = t^2 - 4$

$$\frac{df}{dt} = t^2 - 4 \quad d/dx\,(x^n) = nx^{n-1}$$

$$\frac{df}{dt} = 2t$$

$\therefore$ Velocity vector $= i + 2tj$

at $t=1 \longrightarrow i + 2(1)j = i + 2j$

the acceleration of the vector $a(t)$ is the derivative of the velocity vector $v(t)$.

$a(t) = \frac{dv}{dt}$ $\quad v(t) = i + 2t\,j$

$a(t) = 2j \rightarrow$ the acceleration does not depend on t.

the particle follows the path described by the equation $x^2 + 10x + 21$.
At $t=1$, the velocity is $i + 2j$ and the acceleration $2j$.

$\therefore$ accel. at $t=1$ is $2j$.

The position of a particle in the xy-plane at time t is $r(t) = (e^t)\hat{i} + \left(\frac{5}{36}e^{2t}\right)\hat{j}$

Find an equation in x and y whose graph is the path of the particle. Then find the particle's velocity and acceleration vectors at $t = \ln 6$.

COMMON FACTORING EXAMPLES
$x^2 - a^2 = (x+a)(x-a)$
$x^2 + 2ax + a^2 = (x+a)^2$
$x^2 - 2ax + a^2 = (x-a)^2$
$x^2 + (a+b)x + ab = (x+a)(x+b)$
$x^3 + 3ax^2 + 3a^2x + a^3 = (x+a)^3$
$x^3 + a^3 = (x+a)(x^2 - ax + a^2)$
$x^3 - a^3 = (x-a)(x^2 + ax + a^2)$
$x^{2n} - a^{2n} = (x^n - a^n)(x^n + a^n)$

COMMON DERIVATIVES
$\frac{d}{dx}(x) = 1$
$\frac{d}{dx}(\sin x) = \cos x$
$\frac{d}{dx}(\cos x) = -\sin x$
$\frac{d}{dx}(\tan x) = \sec^2 x$
$\frac{d}{dx}(\sec x) = \sec x \tan x$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$
$\frac{d}{dx}(\cot x) = -\csc^2 x$
$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
$\frac{d}{dx}(a^x) = a^x \ln(a)$
$\frac{d}{dx}(e^x) = e^x$
$\frac{d}{dx}(\ln(x)) = \frac{1}{x}, x > 0$
$\frac{d}{dx}(\ln\lvert x\rvert) = \frac{1}{x}$
$\frac{d}{dx}(\log_a(x)) = \frac{1}{x\ln(a)}$

given that $r(t) = (e^t)\hat{i} + \left(\frac{5}{36}e^{2t}\right)\hat{j}$

then $r(t) = x(t)\hat{i} + y(t)\hat{j}$

$$\boxed{x(t) = e^t}$$

$$y(t) = \frac{5}{36}e^{2t}$$

$$y(t) = \frac{5}{36}(e^t)^2$$

$$= \frac{5}{36}(x(t))^2 \quad \text{since } x(t) = e^t$$

$$\boxed{y(t) = \frac{5}{36}x^2} \quad \text{equation for the path}$$

Velocity of the particle

$$v(t) = \frac{dr}{dt} = \frac{df}{dt}i = \frac{dg}{dt}j$$

$$= \left(\frac{d}{dt}e^t\right)\hat{i} + \left(\frac{d}{dt}\frac{5}{36}e^{2t}\right)\hat{j}$$

$$= e^t\hat{i} + \frac{5}{36}(2e^{2t})\hat{j} \quad d/dx(e^x) = e^x \quad d/dx(x^n) = nx^{n-1}$$

$$= e^t\hat{i} + \frac{5}{18}e^{2t}\hat{j}$$

the acceleration of the particle

$$a(t) = \frac{dv}{dt} = \frac{d}{dt}\left(e^t\hat{i} + \frac{5}{18}e^{2t}\hat{j}\right)$$

$$= \frac{d}{dt}(e^t)\hat{i} + \frac{5}{18}\left(\frac{d}{dt}e^{2t}\right)\hat{j}$$

$$= e^t\hat{i} + \frac{5}{18}(2e^{2t})\hat{j} \quad \frac{d}{dt}e^{at} = ae^{at}$$

$$= e^t\hat{i} + \frac{5}{9}e^{2t}\hat{j}$$

Velocity of the particle at $t = \ln 6$

$$v(\ln 6) = e^t\hat{i} + \frac{5}{18}e^{2t}\hat{j}$$

$$= e^{\ln 6}\hat{i} + \frac{5}{18}e^{2\cdot\ln 6}\hat{j}$$

$$= e^{\ln 6}\hat{i} + \frac{5}{18}e^{\ln 6^2}\hat{j}$$

$$= e^{\ln 6}\hat{i} + \frac{5}{18}\cdot 36\,\hat{j} \quad e^{\ln x} = x$$

$$= 6\hat{i} + 10\hat{j}$$

the acceleration of the particle at $t = \ln 6$

$$a(\ln 6) = e^t\hat{i} + \frac{5}{9}e^{2t}\hat{j}$$

$$= e^{\ln 6}\hat{i} + \frac{5}{9}e^{2\ln 6}\hat{j}$$

$$= e^{\ln 6}\hat{i} + \frac{5}{9}e^{\ln 6^2}\hat{j}$$

$$e^{\ln x} = x \quad = e^{\ln 6}\hat{i} + \frac{5}{9}\cdot 36\,\hat{j}$$

$$= 6\hat{i} + 20\hat{j}$$

why $e^{2\ln 6} = e^{\ln 6^2}$

The path $r(t) = (2\sin t)\hat{i} + (2\cos t)\hat{j}$ describes the motion on the circle $x^2 + y^2 = 4$. Find the particle's velocity and acceleration vectors at $t = \frac{\pi}{4}$ and $\frac{\pi}{6}$, and sketch them as vectors on the curve.

① Find an expression for the velocity as a function of time.

Velocity of the vector $v(t)$ is the derivative of the position vector $r(t)$

$$v(t) = \frac{dr}{dt} = \frac{df}{dt} i = \frac{dg}{dt} j$$

$$v(t) = \frac{df}{dt}(2\sin t)\hat{i} + \frac{dg}{dt}(2\cos t)\hat{j}$$

$$v(t) = (2\cos t)\hat{i} + (-2\sin t)\hat{j}$$

velocity vector function

② Find an expression for the acceleration as a function of time.

$$a(t) = \frac{dv}{dt} \qquad v(t) = (2\cos t)\hat{i} + (-2\sin t)\hat{j}$$

$$a(t) = (-2\sin t)\hat{i} + (-2\cos t)\hat{j}$$

acceleration vector function

VELOCITY AT $t\ \frac{\pi}{4}$

$$v\left(\frac{\pi}{4}\right) = \left(2\cos\frac{\pi}{4}\right)\hat{i} + \left(-2\sin\frac{\pi}{4}\right)\hat{j}$$

$$= \left(2\cdot\frac{1}{\sqrt{2}}\right)\hat{i} + \left(-2\cdot\frac{1}{\sqrt{2}}\right)\hat{j}$$

$$= \frac{2}{\sqrt{2}}\hat{i} + -\frac{2}{\sqrt{2}}\hat{j}$$

$$= \frac{2}{2^{\frac{1}{2}}}\hat{i} + \frac{-2}{2^{\frac{1}{2}}}\hat{j} \qquad \sqrt[n]{a} = a^{\frac{1}{n}}$$

$$= 2^{1-\frac{1}{2}}\hat{i} + -2^{1-\frac{1}{2}}\hat{j} \qquad \frac{x^a}{x^b} = x^{a-b}$$

$$= 2^{\frac{1}{2}}\hat{i} + -2^{\frac{1}{2}}\hat{j}$$

$$v\left(\frac{\pi}{4}\right) = \sqrt{2}\,\hat{i} + (-\sqrt{2})\hat{j} \qquad \sqrt[n]{a} = a^{\frac{1}{n}}$$

VELOCITY AT $t\ \frac{\pi}{6}$

$$v\left(\frac{\pi}{6}\right) = \left(2\cos\frac{\pi}{6}\right)\hat{i} + \left(-2\sin\frac{\pi}{6}\right)\hat{j}$$

$$= \left(2\cdot\frac{\sqrt{3}}{2}\right)\hat{i} + \left(-2\cdot\frac{1}{2}\right)\hat{j}$$

$$v\left(\frac{\pi}{6}\right) = \sqrt{3}\,\hat{i} + (-1)\hat{j}$$

ACCELERATION AT $t\ \frac{\pi}{4}$

$$a\left(\frac{\pi}{4}\right) = \left(-2\sin\frac{\pi}{4}\right)\hat{i} + \left(-2\cos\frac{\pi}{4}\right)\hat{j}$$

$$= \left(-2\cdot\frac{1}{\sqrt{2}}\right)\hat{i} + \left(-2\cdot\frac{1}{\sqrt{2}}\right)$$

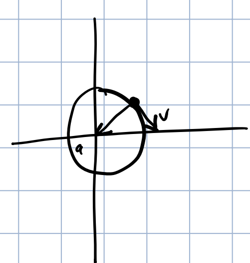
$$a\left(\frac{\pi}{4}\right) = -\sqrt{2}\,\hat{i} + (-\sqrt{2})\hat{j}$$

ACCELERATION AT $t\ \frac{\pi}{6}$

$$a\left(\frac{\pi}{6}\right) = \left(-2\sin\frac{\pi}{6}\right)\hat{i} + \left(-2\cos\frac{\pi}{6}\right)\hat{j}$$

$$= \left(-2\cdot\frac{1}{2}\right)\hat{i} + \left(-2\cdot\frac{\sqrt{3}}{2}\right)\hat{j}$$

$$= -1\hat{i} + \sqrt{3}\,\hat{j}$$



The path $r(t) = (3 \sin t)\hat{i} + (3 \cos t)\hat{j}$ describes the motion on the circle $x^2 + y^2 = 9$. Find the particle's velocity and acceleration vectors at $t = \frac{\pi}{2}$ and $\frac{\pi}{4}$, and sketch them as vectors on the curve.

$$r(t) = (3 \sin t)\hat{i} + (3 \cos t)\hat{j}$$

Velocity $\vec{v}(t) = \vec{r}\,'(t)$

$$\vec{v}(t) = \frac{d}{dt}(3 \sin t)\hat{i} + \frac{d}{dt}(3 \cos t)\hat{j}$$

$$\boxed{\vec{v}(t) = (3 \cos t)\hat{i} + (-3 \sin t)\hat{j}}$$

Acceleration $a(t) = \vec{v}\,'(t)$

$$\vec{a}(t) = (3 \cos t)\hat{i} + (-3 \sin t)\hat{j}$$

$$\boxed{\vec{a}(t) = (-3 \sin t)\hat{i} + (-3 \cos t)\hat{j}}$$

Velocity at $t = \frac{\pi}{2}$

$$\vec{v}\left(\tfrac{\pi}{2}\right) = \left(3 \cos \tfrac{\pi}{2}\right)\hat{i} + \left(-3 \sin \tfrac{\pi}{2}\right)\hat{j}$$

$$= (3 \cdot 0)\hat{i} + (-3 \cdot 1)\hat{j}$$

$$\boxed{\vec{v}\left(\tfrac{\pi}{2}\right) = -3\hat{j}}$$

Velocity at $t = \frac{\pi}{4}$

$$\vec{v}\left(\tfrac{\pi}{4}\right) = \left(3 \cos \tfrac{\pi}{4}\right)\hat{i} + \left(-3 \sin \tfrac{\pi}{4}\right)\hat{j}$$

$$= \left(3 \cdot \tfrac{\sqrt{2}}{2}\right)\hat{i} + \left(-3 \cdot \tfrac{\sqrt{2}}{2}\right)\hat{j}$$

$$\boxed{\vec{v}\left(\tfrac{\pi}{4}\right) = \tfrac{3\sqrt{2}}{2}\hat{i} + \tfrac{-3\sqrt{2}}{2}\hat{j}}$$

Acceleration at $t = \frac{\pi}{2}$

$$\vec{a}\left(\tfrac{\pi}{2}\right) = \left(-3 \sin \tfrac{\pi}{2}\right)\hat{i} + \left(-3 \cos \tfrac{\pi}{2}\right)\hat{j}$$

$$= (-3 \cdot 1)\hat{i} + (-3 \cdot 0)\hat{j}$$

$$\boxed{\vec{a}\left(\tfrac{\pi}{2}\right) = -3\hat{i}}$$

Acceleration at $t = \frac{\pi}{4}$

$$\vec{a}\left(\tfrac{\pi}{4}\right) = \left(-3 \sin \tfrac{\pi}{4}\right)\hat{i} + \left(-3 \cos \tfrac{\pi}{4}\right)\hat{j}$$

$$= \left(-3 \cdot \tfrac{\sqrt{2}}{2}\right)\hat{i} + \left(-3 \cdot \tfrac{\sqrt{2}}{2}\right)\hat{j}$$

$$\boxed{\vec{a}\left(\tfrac{\pi}{4}\right) = -\tfrac{3\sqrt{2}}{2}\hat{i} + -\tfrac{3\sqrt{2}}{2}\hat{j}}$$

Position of the particle at $t = \frac{\pi}{2}$

$$\vec{r}\left(\tfrac{\pi}{2}\right) = (3 \sin t)\hat{i} + (3 \cos t)\hat{j}$$

$$= \left(3 \sin \tfrac{\pi}{2}\right)\hat{i} + \left(3 \cos \tfrac{\pi}{2}\right)\hat{j}$$

$$= (3 \cdot 1)\hat{i} + (3 \cdot 0)\hat{j}$$

$$\boxed{\vec{r}\left(\tfrac{\pi}{2}\right) = 3\hat{i}}$$

Position of the particle at $t = \frac{\pi}{4}$

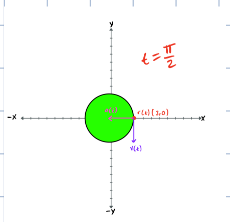
$$\vec{r}\left(\tfrac{\pi}{4}\right) = (3 \sin t)\hat{i} + (3 \cos t)\hat{j}$$

$$= \left(3 \sin \tfrac{\pi}{4}\right)\hat{i} + \left(3 \cos \tfrac{\pi}{4}\right)\hat{j}$$

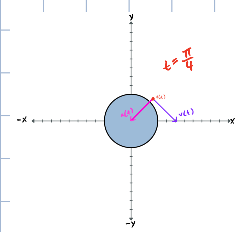
$$= \left(3 \cdot \tfrac{\sqrt{2}}{2}\right)\hat{i} + \left(3 \cdot \tfrac{\sqrt{2}}{2}\right)\hat{j}$$

$$\boxed{\vec{r}\left(\tfrac{\pi}{4}\right) = \tfrac{3\sqrt{2}}{2}\hat{i} + \tfrac{3\sqrt{2}}{2}\hat{j}}$$

At $t = \frac{\pi}{2}$,
the particle's position is $3\hat{i} + 0\hat{j}$,
its velocity is $0\hat{i} + (-3)\hat{j}$
and its acceleration $-3\hat{i} + 0\hat{j}$.



At $t = \frac{\pi}{4}$
the particle's position is $\frac{3\sqrt{2}}{2}\hat{i} + \frac{3\sqrt{2}}{2}\hat{j}$
its velocity is $\frac{3\sqrt{2}}{2}\hat{i} + \frac{-3\sqrt{2}}{2}\hat{j}$
and its acceleration $-\frac{3\sqrt{2}}{2}\hat{i} + -\frac{3\sqrt{2}}{2}\hat{j}$



the path $r(t) = (t - \sin t)\hat{i} + (1 - \cos t)\hat{j}$ describes the motion on the circle $x = t - \sin t$, $y = 1 - \cos t$. Find the particle's velocity and acceleration vectors at $t = \pi$, and sketch them as vectors on the curve.

$r(t) = (t - \sin t)\hat{i} + (1 - \cos t)\hat{j}$

→ Velocity $\vec{v}(t) = \vec{r}\,'(t)$

$$\vec{v}(t) = \frac{d}{dt}(t - \sin t)\hat{i} + \frac{d}{dt}(1 - \cos t)\hat{j}$$

$$= (1 - \cos t)\hat{i} + (0 - (-\sin t))\hat{j}$$

$$\boxed{\vec{v}(t) = (1 - \cos t)\hat{i} + (\sin t)\hat{j}}$$

→ Acceleration $a(t) = \vec{v}\,'(t)$

$$\vec{a}(t) = (1 - \cos t)\hat{i} + (\sin t)\hat{j}$$

$$= (0 - (-\sin t))\hat{i} + \cos t\,\hat{j}$$

$$\boxed{\vec{a}(t) = \sin t\,\hat{i} + \cos t\,\hat{j}}$$

→ Velocity at $t = \pi$

$$\vec{v}(\pi) = (1 - \cos\pi)\hat{i} + (\sin\pi)\hat{j}$$

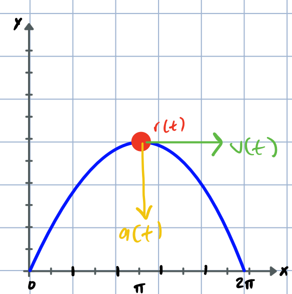
$$= (1 - (-1))\hat{i} + 0\hat{j}$$

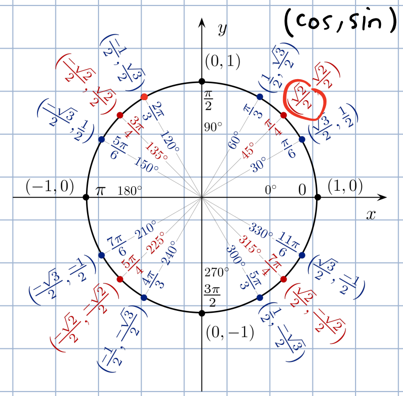
$$\boxed{\vec{v}(\pi) = 2\hat{i} + 0\hat{j}}$$

→ Acceleration at $t = \pi$

$$\vec{a}(\pi) = \sin\pi\,\hat{i} + \cos\pi\,\hat{j}$$

$$\boxed{\vec{a}(\pi) = 0\hat{i} + (-1)\hat{j}}$$





The equation $\vec{r}(t) = (3t+6)\hat{i} + (7t^2-6)\hat{j} + (4t)\hat{k}$ is the position of a particle in space at time t. Find the particle's velocity and acceleration vectors. then write the particle's velocity at $t=0$ as a product of its speed and direction.

$\vec{r}(t) = (3t+6)\hat{i} + (7t^2-6)\hat{j} + (4t)\hat{k}$, $t = 0$.

→ Velocity $\vec{v}(t) = \vec{r}\,'(t)$

$$\vec{v}(t) = \frac{d}{dt}(3t+6)\hat{i} + \frac{d}{dt}(7t^2-6)\hat{j} + \frac{d}{dt}(4t)\hat{k}$$

$$\boxed{\vec{v}(t) = 3\hat{i} + 14t\hat{j} + 4\hat{k}}$$

→ Acceleration $a(t) = \vec{v}\,'(t)$

$$\vec{a}(t) = 3\hat{i} + 14t\hat{j} + 4\hat{k}$$

$$\boxed{\vec{a}(t) = 0\hat{i} + 14\hat{j} + 0\hat{k}}$$

→ speed $|\vec{v}(t)|$ At $t=0$

$$|\vec{v}(t)| = \sqrt{3^2 + (14t)^2 + 4^2}$$

$$= \sqrt{9 + 196(0)^2 + 16}$$

$$= \sqrt{9 + 16}$$

$$= \sqrt{25}$$

$$\boxed{|\vec{v}(t)| = 5}$$

→ direction $\frac{\vec{v}}{|\vec{v}|}$ At $t=0$

$$\frac{\vec{v}}{|\vec{v}|} = \frac{3\hat{i} + 14t\hat{j} + 4\hat{k}}{5}$$

$$\frac{\vec{v}}{|\vec{v}|} = \frac{3\hat{i} + 14(0)\hat{j} + 4\hat{k}}{5}$$

$$\boxed{\frac{\vec{v}}{|\vec{v}|} = \frac{3\hat{i}}{5} + \frac{0}{5}\hat{j} + \frac{4\hat{k}}{5}}$$

→ velocity of the particle at $t=0$ as a product of speed and direction

$$\boxed{v(0) = 5\left(\frac{3\hat{i}}{5} + \frac{0}{5}\hat{j} + \frac{4\hat{k}}{5}\right)}$$

The equation $\vec{r}(t) = (\sin t)\hat{i} + (\cos t)\hat{j} + (t)\hat{k}$ is the position of a particle in space at time t. Find the particle's velocity and acceleration vectors. Then write the particle's velocity at $t = \frac{\pi}{2}$ as a product of its speed and direction.

$$\vec{r}(t) = (\sin t)\hat{i} + (\cos t)\hat{j} + (t)\hat{k}, \quad t = \frac{\pi}{2}$$

→ Velocity $\vec{v}(t) = \vec{r}\,'(t)$

$$\vec{v}(t) = \frac{d}{dt}(\sin t)\hat{i} + \frac{d}{dt}(\cos t)\hat{j} + \frac{d}{dt}(t)\hat{k}$$

$$\boxed{\vec{v}(t) = (\cos t)\hat{i} + (-\sin t)\hat{j} + 1\hat{k}}$$

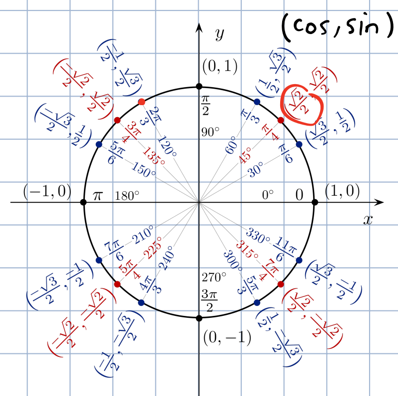
COMMON DERIVATIVES
$\frac{d}{dx}(x) = 1$
$\frac{d}{dx}(\sin x) = \cos x$
$\frac{d}{dx}(\cos x) = -\sin x$
$\frac{d}{dx}(\tan x) = \sec^2 x$
$\frac{d}{dx}(\sec x) = \sec x \tan x$
$\frac{d}{dx}(\csc x) = -\csc x \cot x$
$\frac{d}{dx}(\cot x) = -\csc^2 x$
$\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$
$\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
$\frac{d}{dx}(a^x) = a^x \ln(a)$
$\frac{d}{dx}(e^x) = e^x$
$\frac{d}{dx}(\ln(x)) = \frac{1}{x}, x > 0$
$\frac{d}{dx}(\ln\lvert x\rvert) = \frac{1}{x}$
$\frac{d}{dx}(\log_a(x)) = \frac{1}{x\ln(a)}$

→ Acceleration $a(t) = \vec{v}\,'(t)$

$$\vec{a}(t) = (\cos t)\hat{i} + (-\sin t)\hat{j} + 1\hat{k}$$

$$\boxed{\vec{a}(t) = (-\sin t)\hat{i} + (-\cos t)\hat{j} + 0\hat{k}}$$

→ speed $|\vec{v}(t)|$ At $t = \frac{\pi}{2}$



$$|\vec{v}(t)| = \sqrt{(\cos t)^2 + (-\sin t)^2 + 1^2}$$

$$= \sqrt{\cos^2 t + \sin^2 t + 1}$$

$$= \sqrt{\cos^2 \tfrac{\pi}{2} + \sin^2 \tfrac{\pi}{2} + 1}$$

$$= \sqrt{0 + 1 + 1}$$

$$\boxed{|\vec{v}(t)| = \sqrt{2}}$$

→ direction $\frac{\vec{v}}{|\vec{v}|}$ At $t = \frac{\pi}{2}$

$$\frac{\vec{v}}{|\vec{v}|} = \frac{(\cos t)\hat{i} + (-\sin t)\hat{j} + 1\hat{k}}{\sqrt{2}}$$

$$\frac{\vec{v}}{|\vec{v}|} = \frac{\cos(\frac{\pi}{2})\hat{i}}{\sqrt{2}} + \frac{(-\sin\frac{\pi}{2})\hat{j}}{\sqrt{2}} + \frac{1\hat{k}}{\sqrt{2}}$$

$$\frac{\vec{v}}{|\vec{v}|} = \frac{0}{\sqrt{2}}\hat{i} + \frac{(-1)}{\sqrt{2}}\hat{j} + \frac{1}{\sqrt{2}}\hat{k}$$

$$\frac{\vec{v}}{|\vec{v}|} = 0\hat{i} + \frac{(-1)}{\sqrt{2}}\hat{j} \cdot \sqrt{2} + \frac{1}{\sqrt{2}}\hat{k} \cdot \sqrt{2}$$

$$\boxed{\frac{\vec{v}}{|\vec{v}|} = 0\hat{i} + \frac{-\sqrt{2}}{2}\hat{j} + \frac{\sqrt{2}}{2}\hat{k}}$$

$$\boxed{\therefore\ v\left(\tfrac{\pi}{2}\right) = \sqrt{2}\left(0\hat{i} + \frac{-\sqrt{2}}{2}\hat{j} + \frac{\sqrt{2}}{2}\hat{k}\right)}$$

The equation $\vec{r}(t) = (t+2)\hat{i} + (\sqrt{3}\,t)\hat{j} + (2t^2)k$ is the position of a particle in space at time t. Find the angle between the velocity and acceleration vectors at time $t = 0$.

$\vec{r}(t) = (t+2)\hat{i} + (\sqrt{3}\,t)\hat{j} + (2t^2)k \;,\quad t = 0.$

→ Velocity $\vec{v}(t) = \vec{r}\,'(t)$

$$\vec{v}(t) = \frac{d}{dt}(t+2)\hat{i} + \frac{d}{dt}(\sqrt{3}\,t)\hat{j} + \frac{d}{dt}(2t^2)\hat{k}$$

$$\boxed{\vec{v}(t) = 1\hat{i} + \sqrt{3}\,\hat{j} + 4t\,\hat{k}}$$

$$\vec{v}(0) = 1\hat{i} + \sqrt{3}\,\hat{j} + 4(0)\hat{k}$$

$$\boxed{\vec{v}(0) = 1\hat{i} + \sqrt{3}\,\hat{j} + 0\hat{k}}$$

→ Acceleration $a(t) = \vec{v}\,'(t)$

$$\vec{a}(t) = 1\hat{i} + \sqrt{3}\,\hat{j} + 4t\,\hat{k}$$

$$\boxed{\vec{a}(t) = 0\hat{i} + 0\hat{j} + 4\hat{k}}$$

acceleration does not depend on t.

→ Acceleration $\theta = \cos^{-1}\dfrac{\vec{u}\cdot\vec{v}}{|\vec{u}||\vec{v}|}$

Find dot product of $\vec{v} = 1\hat{i} + \sqrt{3}\,\hat{j} + 0\hat{k}$ and $\vec{a} = 0\hat{i} + 0\hat{j} + 4\hat{k}$

$$\begin{aligned}\vec{v}\cdot\vec{a} &= (1)(0) + (\sqrt{3})(0) + (0)(4)\\ &= 0\end{aligned}$$

since $\vec{v}\cdot\vec{a} = 0$, the magnitudes do not need to be evaluated.

Find angle $\theta = \cos^{-1}(0)$

$$\boxed{\theta = \frac{\pi}{2} \text{ radians}}$$

Find parametric equations for the line that is tangent to the given curve at the given parameter value. $\vec{r}(t) = (3\sin t)\,i + (t^2 - \cos t)\hat{j} + (4e^t)k \;,\quad t = 0.$

$$\vec{r}(t) = (3\sin t)\hat{i} + (t^2 - \cos t)\hat{j} + (4e^t)\hat{k}$$

$$\vec{r}(0) = (3\sin(0))\hat{i} + (0^2 - \cos(0))\hat{j} + (4e^0)\hat{k}$$

$$\vec{r}(0) = 0\hat{i} - 1\hat{j} + 4\hat{k}$$

$$\vec{v}(t) = \frac{dr}{dt}(3\sin t)\hat{i} + \frac{dr}{dt}(t^2 - \cos t)\hat{j} + \frac{dr}{dt}(4e^t)k$$

$$= (3\cos t)\hat{i} + (2t + \sin t)\hat{j} + (4e^t)k$$

$$\vec{v}(0) = (3\cos(0))\hat{i} + (2(0) + \sin(0))\hat{j} + 4e^{(0)}k$$

$$= 3\hat{i} + 0\hat{j} + 4k$$

therefore, the tangent passes through the point $(0,-1,4)$. and it's parallel to $3\hat{i} + 0\hat{j} + 4k$.

$$\begin{aligned}x &= (0) + (3)t = 3t\\ y &= (-1) + (0)t = -1\\ z &= (4) + (4)t = 4 + 4t\end{aligned}$$

The equation **r**(t) = **sin**(3t) **i** + **cos**(3t) **j**, t ≥ 0 describes the motion of a particle moving along the unit circle. Answer the following questions about the behavior of the particle.
a. Does the particle have constant speed? If so, what is its constant speed?
b. Is the particle's acceleration vector always orthogonal to its velocity vector?
c. Does the particle move clockwise or counterclockwise around the circle?
d. Does the particle begin at the point (1,0)?

COMMON DERIVATIVES

- $\frac{d}{dx}(x) = 1$
- $\frac{d}{dx}(\sin x) = \cos x$
- $\frac{d}{dx}(\cos x) = -\sin x$
- $\frac{d}{dx}(\tan x) = \sec^2 x$
- $\frac{d}{dx}(\sec x) = \sec x \tan x$
- $\frac{d}{dx}(\csc x) = -\csc x \cot x$
- $\frac{d}{dx}(\cot x) = -\csc^2 x$
- $\frac{d}{dx}(\sin^{-1} x) = \frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}(\cos^{-1} x) = -\frac{1}{\sqrt{1-x^2}}$
- $\frac{d}{dx}(\tan^{-1} x) = \frac{1}{1+x^2}$
- $\frac{d}{dx}(a^x) = a^x \ln(a)$
- $\frac{d}{dx}(e^x) = e^x$
- $\frac{d}{dx}(\ln(x)) = \frac{1}{x}, x > 0$
- $\frac{d}{dx}(\ln|x|) = \frac{1}{x}$
- $\frac{d}{dx}(\log_a(x)) = \frac{1}{x\ln(a)}$

a) $\vec{r}(t) = \sin(3t)\,\hat{i} + \cos(3t)\,\hat{j}, \quad t \geq 0.$

$\vec{v}(t) = \frac{dr}{dt}(\sin(3t))\,\hat{i} + \frac{dr}{dt}(\cos(3t))\,\hat{j}$ Chain rule $F'(g(x)) = F'(g(x))\,g'(x)$

$\vec{v}(t) = \cos(3t)(3)\,\hat{i} + -\sin(3t)(3)\,\hat{j}$

$\vec{v}(t) = 3\cos(3t)\,\hat{i} + (-3\sin(3t))\,\hat{j}$

$\vec{a}(t) = \frac{dv}{dt}\,3\cos(3t)\,\hat{i} + \frac{dv}{dt}(-3\sin(3t))\,\hat{j}$

$= 3\,\frac{dv}{dt}\cos(3t)\,\hat{i} + (-3)\,\frac{dv}{dt}\sin(3t)\,\hat{j}$

$= -3\cdot\sin(3t)(3)\,\hat{i} + (-3\cos(3t)(3))\,\hat{j}$

$\vec{a}(t) = -9\sin(3t)\,\hat{i} + (-9\cos(3t))\,\hat{j}$

Speed $|\vec{v}(t)| = \sqrt{(3\cos(3t))^2 + (-3\sin(3t))^2}$

$|\vec{v}(t)| = \sqrt{9\cos^2(3t) + 9\sin^2(3t)}$

$|\vec{v}(t)| = \sqrt{9(\cos^2(3t) + \sin^2(3t))}$ Factor 9. $\cos^2\theta + \sin^2\theta = 1$

$|\vec{v}(t)| = \sqrt{9}$

$|\vec{v}(t)| = 3$ therefore the particle's constant speed is 3.

b) $\vec{v}\cdot\vec{a} = [3\cos(3t)\,\hat{i} + (-3\sin(3t))\,\hat{j}] + [-9\sin(3t)\,\hat{i} + (-9\cos(3t))\,\hat{j}]$

$\vec{v}\cdot\vec{a} = (-27\cos(3t)\cdot\sin(3t)) + (27\sin(3t)\cdot\cos(3t))$

$\vec{v}\cdot\vec{a} = 0$ therefore, acceleration vector is orthogonal to velocity vector.

c) set $t=0$ in $\vec{r}(t) = \sin(3t)\,\hat{i} + \cos(3t)\,\hat{j}$

$\vec{r}(0) = \sin(3(0))\,\hat{i} + \cos(3(0))\,\hat{j}$

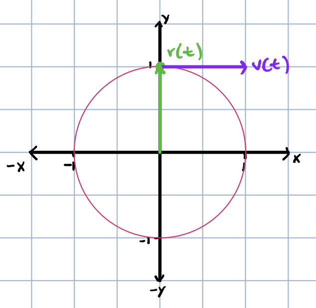
$\vec{r}(0) = 0\,\hat{i} + 1\,\hat{j}$

$\vec{v}(t) = 3\cos(3t)\,\hat{i} + (-3\sin(3t))\,\hat{j}$

$\vec{v}(t) = 3\cos(3(0))\,\hat{i} + (-3\sin(3(0)))\,\hat{j}$

$\vec{v}(t) = 3\cdot 1\,\hat{i} + 0\,\hat{j}$

$\vec{v}(t) = 3\,\hat{i} + 0\,\hat{j}$



Evaluate the integral $\int_0^1 [(5t^3)\,\hat{i} + (2)\,\hat{j} + (3t+6)\,\hat{k}]\,dt$

$\int_0^1 5t^3\,\hat{i}\,dt + \int_0^1 2\,\hat{j}\,dt + \int_0^1 3t+6\,\hat{k}\,dt$

$= 5\cdot\frac{t^4}{4}\Big|_0^1\,\hat{i} + 2t\Big|_0^1\,\hat{j} + 3\cdot\frac{t^2}{2} + 6t\Big|_0^1$ $\int x^a\,dx = \frac{x^{a+1}}{a+1}$ $\int a\,dx = ax$

$= \frac{5(1)^4}{4}\,\hat{i} + 2(1)\,\hat{j} + \frac{3(1)^2}{2} + 6(1)\,k$

$= \frac{5}{4}\,\hat{i} + 2\,\hat{j} + \frac{15}{2}\,\hat{k}$

Evaluate the integral $\int_0^1 [(9te^{6t^2})\hat{i} + (5e^{-t})\hat{j} + (8)\hat{k}]\,dt$

$$\int_0^1 (9te^{6t^2})\hat{i}\,dt + \int_0^1 (5e^{-t})\hat{j}\,dt + \int_0^1 (8)\hat{k}\,dt$$

adjust integral boundaries

$t=0 \rightarrow u=0$ $\quad u = 6t^2 = 6\cdot 0^2 = 0$

$t=1 \rightarrow u=6$ $\quad u = 6t^2 = 6\cdot 1^2 = 6$

$t=0 \rightarrow u=0$ $\quad u = -t = -0 = 0$

$t=1 \rightarrow u=-1$ $\quad u = -t = -1$

$u = 6t^2$ $\qquad u = -t$

$\frac{du}{dt} = 12t$ $\qquad \frac{du}{dt} = -1$

$du = 12t\,dt$ $\qquad du = -1\,dt$

$dt = \frac{1}{12t}du$ $\qquad dt = -1\,du$

$$9\int_0^1 \cancel{t}e^u \frac{1}{12\cancel{t}}du + 5\int_0^1 e^{-t}dt$$

$$9\int_0^1 \frac{e^u}{12}du + 5 - \int_0^1 -e^u\,du$$

$$9\cdot\frac{1}{12}\int_0^1 e^u\,du + 5(e^u)$$

substitute u back $\rightarrow$ $\frac{3}{4}e^u + (5e^u)$

$\int e^u\,du = e^u$ $\quad \frac{3}{4}e^u\Big|_0^6\,\hat{i} + 5e^u\Big|_{-1}^0\,\hat{j} + 8\hat{k}$

$$\frac{3}{4}(e^6 - e^0) + 5(e^0 - e^{-1})$$

$(e^u) = \frac{1}{e}$ $\quad e^0 = 1$ $\quad \frac{3(e^6-1)}{4}\hat{i} + 5\left(1-\frac{1}{e}\right) + 8\hat{k}$

solve the initial value problem for $\vec{r}$ as a vector function of t.

Differential equation : $\frac{dr}{dt} = -t\hat{i} - 3t\hat{j} - 5t\,k$ $\qquad$ Initial conditions : $\vec{r}(0) = 7\hat{i} + 8\hat{j} + \hat{k}$

① $\vec{r}(t) = \int(-t\hat{i} - 3t\hat{j} - 5t\,k)\,dt$

$$\vec{r}(t) = \int(-t)\,dt\,\hat{i} + \int(-3t)\,dt\,\hat{j} + \int(-5t)\,dt\,\hat{k}$$

$$\boxed{\vec{r}(t) = -\frac{t^2}{2}\hat{i} + \left(-\frac{3t^2}{2}\right)\hat{j} + \left(-\frac{5t^2}{2}\right)\hat{k} + C}$$

Vector function

Using the initial condition $\vec{r}(0) = 7\hat{i} + 8\hat{j} + \hat{k}$ to find the value of C.

② Substitute $t = 0$ in $\vec{r}(t)$

$$\vec{r}(t) = -\frac{t^2}{2}\hat{i} + \left(-\frac{3t^2}{2}\right)\hat{j} + \left(-\frac{5t^2}{2}\right)\hat{k} + C$$

$$\vec{r}(0) = -\frac{0^2}{2}\hat{i} + \left(-\frac{3\cdot 0^2}{2}\right)\hat{j} + \left(-\frac{5\cdot 0^2}{2}\right)\hat{k} + C$$

$$7\hat{i} + 8\hat{j} + \hat{k} = 0\hat{i} + 0\hat{j} + 0k + C$$

$$\boxed{C = 7\hat{i} + 8\hat{j} + \hat{k}}$$

③ Substitute $C = 7\hat{i} + 8\hat{j} + \hat{k}$ in $\vec{r}(t)$

$$\vec{r}(t) = -\frac{t^2}{2}\hat{i} + \left(-\frac{3t^2}{2}\right)\hat{j} + \left(-\frac{5t^2}{2}\right)\hat{k} + 7\hat{i} + 8\hat{j} + \hat{k}$$

$$\boxed{\vec{r}(t) = \left(7 - \frac{t^2}{2}\right)\hat{i} + \left(8 - \frac{3t^2}{2}\right)\hat{j} + \left(1 - \frac{5t^2}{2}\right)\hat{k}}$$

Solve the initial value problem for $\vec{r}$ as a vector function of t.

Differential equation : $\dfrac{dr}{dt} = \dfrac{9}{2}(t+1)^{1/2}\,\hat{\imath} + 2e^{-t}\,\hat{\jmath} + \dfrac{1}{t+1}\,\hat{k}$

Initial conditions : $\vec{r}(0) = \hat{k}$

① $\vec{r}(t) = \displaystyle\int \left(\frac{9}{2}(t+1)^{1/2}\,\hat{\imath} + 2e^{-t}\,\hat{\jmath} + \frac{1}{t+1}\,\hat{k} \right) dt$

$\vec{r}(t) = \displaystyle\int \left(\frac{9}{2}(t+1)^{1/2} dt \right)\hat{\imath} + \int \left(2e^{-t} dt \right)\hat{\jmath} + \int \left(\frac{1}{t+1} dt \right)\hat{k}$

$$\boxed{\vec{r}(t) = 3(t+1)^{3/2}\,\hat{\imath} + (-2e^{-t})\,\hat{\jmath} + \ln|t+1|\,\hat{k} + C}$$

Using the initial condition $\vec{r}(0) = \hat{k}$ to find the value of C.

② Substitute $t = 0$ in $\vec{r}(t)$

$\vec{r}(t) = 3(t+1)^{3/2}\,\hat{\imath} + (-2e^{-t})\,\hat{\jmath} + \ln|t+1|\,\hat{k} + C$

$\vec{r}(0) = 3(0+1)^{3/2}\,\hat{\imath} + (-2e^{-0})\,\hat{\jmath} + \ln|0+1|\,\hat{k} + C$

$\hat{k} = 3\hat{\imath} + (-2)\hat{\jmath} + 0\hat{k} + C$

$C = -3\hat{\imath} + 2\hat{\jmath} - 0\hat{k} + 1\hat{k}$

$C = -3\hat{\imath} + 2\hat{\jmath} + 1\hat{k}$

③ Substitute $C = 3\hat{\imath} + (-2)\hat{\jmath} + 1\hat{k}$ in $\vec{r}(t)$

$\vec{r}(t) = 3(t+1)^{3/2}\,\hat{\imath} + (-2e^{-t})\,\hat{\jmath} + \ln|t+1|\,\hat{k} + -3\hat{\imath} + 2\hat{\jmath} + 1\hat{k}$

$$\boxed{\vec{r}(t) = \left(3(t+1)^{3/2} - 3\right)\hat{\imath} + \left(-2e^{-t} + 2\right)\hat{\jmath} + \left(\ln|t+1| + 1\right)\hat{k}}$$

$\frac{9}{2}\int (t+1)^{1/2}\,dt$ $\quad u = t+1 \quad du = 1\,dt \quad dt = 1\,du$

$\frac{9}{2}\int u^{1/2}\,du$ $\quad \rightarrow \int u^{1/2}\,du$

$\frac{9}{2} \cdot \frac{u^{1/2+1}}{1/2+1}$

$\frac{9}{2} \cdot \frac{u^{3/2}}{3/2}$

$\frac{9}{2} \cdot \frac{2u^{3/2}}{3}$ $\quad \frac{u^{3/2}}{1} \cdot \frac{2}{3}$

$3u^{3/2}$

$3(t+1)^{3/2}$

$u = -t \quad du = -1\,dt$

$dt = -1\,du$

$-\int e^{u}\,du$

$2\int e^{-t}\,dt$

$2 \cdot -1 \int e^{u}\,du$

$-2(e^{u})$

$-2e^{-t}$

$u = t+1 \quad du = 1\,dt$

$dt = 1\,du$

$\int \frac{1}{u}\,du \quad \int \frac{1}{u}\,du = \ln|u|$

$\ln|t+1|$

Solve the initial value problem for $\vec{r}$ as a vector function of t.

Differential equation : $\dfrac{d^2r}{dt^2} = 4e^{t}\hat{\imath} - e^{-t}\hat{\jmath} + 12e^{2t}\hat{k}$

Initial conditions : $\vec{r}(0) = 7\hat{\imath} + 3\hat{\jmath} + 4\hat{k}$

$\left.\dfrac{dr}{dt}\right|_{t=0} = -\hat{\imath} + 7\hat{\jmath}$

if $\vec{R}$ is any anti-derivative of $\vec{r}$ then $\vec{r} = \int \vec{r}(t)\,dt = \vec{R}(t) + C$

$\vec{r} = \int (4e^{t}\hat{\imath} - e^{-t}\hat{\jmath} + 12e^{2t}\hat{k})\,dt$

$\vec{r} = \int (4e^{t}\,dt)\hat{\imath} - \int -(e^{-t}dt)\hat{\jmath} + \int (12e^{2t}dt)\hat{k}$

$$\boxed{\frac{dr}{dt} = (4e^{t} + C_1)\hat{\imath} + (e^{-t} + C_2)\hat{\jmath} + (6e^{2t} + C_3)\hat{k}}$$

$\int e^{2t}\,dt$ $u = 2t$ $du = 2dt$ $dt = \frac{1}{2}du$

$\int e^{u}\frac{1}{2}du$

$12 \cdot \int e^{u}\frac{1}{2}du \rightarrow 12\cdot\frac{1}{2}\int e^{u}du$

$6e^{u}$ $\int e^{u}du = e^{u}$

$6e^{2t} + C$

since $\left.\dfrac{dr}{dt}\right|_{t=0} = -\hat{\imath} + 7\hat{\jmath}$

C_1: $4e^{t} + C_1 = -1$
$4e^{0} + C_1 = -1$
$C_1 = -5$

C_2: $e^{-t} + C_2 = 7$
$e^{-0} + C_2 = 7$
$C_2 = 6$

C_3: $6e^{2t} + C_3 = 0$
$6e^{2\cdot 0} + C_3 = 0$
$C_3 = -6$

thus, $\boxed{\dfrac{dr}{dt} = (4e^{t} - 5)\hat{\imath} + (e^{-t} + 6)\hat{\jmath} + (6e^{2t} - 6)\hat{k}}$

$\int [(4e^{t} - 5)\hat{\imath} + (e^{-t} + 6)\hat{\jmath} + (6e^{2t} - 6)\hat{k}]\,dt$

$= \int (4e^{t} - 5dt)\hat{\imath} + \int (e^{-t} + 6dt)\hat{\jmath} + \int (6e^{2t} - 6dt)\hat{k}$

$= (4e^{t} - 5t + C_1)\hat{\imath} + (-e^{-t} + 6t + C_2)\hat{\jmath} + (3e^{2t} - 6t + C_3)\hat{k}$

$u = -t$
$\frac{du}{dt} = -1$
$du = -1dt$
$dt = -1du$
$\int -e^{u}du$

Find the values of C_1, C_2, C_3 to find the general solution $r(t)$.

$6\int e^{2t}dt$ $u = 2t$ $du = 2dt$ $dt = \frac{1}{2}du$

$6\int e^{u}\cdot\frac{1}{2}du$

$6\cdot\frac{1}{2}\int e^{u}du$ $\int e^{u}du = e^{u}$

$3e^{u} \rightarrow 3e^{2t} + C$

C_1: $4e^{t} - 5t + C_1 = 7$
$4e^{0} - 5(0) + C_1 = 7$
$C_1 = 3$

C_2: $-e^{-t} + 6t + C_2 = 3$
$-e^{0} + 6(0) + C_2 = 3$
$C_2 = 4$

C_3: $3e^{2t} - 6t + C_3 = 4$
$3e^{2\cdot 0} - 6(0) + C_3 = 4$
$C_3 = 1$

therefore, $\boxed{\vec{r}(t) = (4e^{t} - 5t + 3)\hat{\imath} + (-e^{-t} + 6t + 4)\hat{\jmath} + (3e^{2t} - 6t + 1)\hat{k}}$

At time $t = 0$, a particle is located at the point $(8,8,8)$.
It travels in a straight line to the point $(2,2,7)$,
Has speed 7 at $(8,8,8)$ and constant acceleration $-6\hat{i} - 6\hat{j} - \hat{k}$.

Find an equation for the position vector $\vec{r}(t)$ of the particle at time t.

Constant acceleration of the particle

$$a = -6\hat{i} - 6\hat{j} - \hat{k}$$

---

① The acceleration of a particle is:

$$a = \frac{dv}{dt}$$

$$= -6\hat{i} - 6\hat{j} - \hat{k}$$

$$dv = (-6\hat{i} - 6\hat{j} - \hat{k})dt$$

Integrating on both sides:

$$\int dv = \int (-6\hat{i} - 6\hat{j} - \hat{k})dt$$

$$v(t) = (-6t)\hat{i} - (6t)\hat{j} - (t)\hat{k} + C_1 \quad (1)$$

---

② the particle travelling in a straigh line direction from point $(8,8,8)$ towards the point $(2,2,7)$.

$$\text{path} = (2i + 2j + 7k) - (8i + 8j + 8k)$$

$$\vec{v} = -6i - 6j - k$$

---

③ Find the arbitrary constant C_1:
at $t = 0$, speed is 7. Thus, $|\vec{v}| = 7$.

the direction is:

$$\boxed{\frac{\vec{v}}{|\vec{v}|} = \frac{-6i - 6j - k}{\sqrt{(-6)^2 + (-6)^2 + (1)^2}}}$$

substitute the values of $|\vec{v}| = 7$ and $\frac{\vec{v}}{|\vec{v}|} = \frac{-6i - 6j - k}{\sqrt{(-6)^2 + (-6)^2 + (1)^2}}$

in the Velocity $= |\vec{v}| \left(\frac{\vec{v}}{|\vec{v}|} \right)$.

that implies,

$$\boxed{\text{Velocity} = |\vec{v}| \left(\frac{\vec{v}}{|\vec{v}|} \right) \rightarrow 7 \cdot \frac{-6i - 6j - k}{\sqrt{(-6)^2 + (-6)^2 + (1)^2}}} \quad (a)$$

From (1)

$$v(t) = (-6t)\hat{i} - (6t)\hat{j} - (t)\hat{k} + C_1$$

At $t = 0$,

$$v(0) = (-6 \cdot 0)\hat{i} - (6 \cdot 0)\hat{j} - (0)\hat{k} + C_1$$

$$v(0) = C_1 \quad (b)$$

from the equations (a) and (b).

$$C_1 = 7 \cdot \frac{-6i - 6j - k}{\sqrt{(-6)^2 + (-6)^2 + (1)^2}}$$

substitute in (1)

$$v(t) = (-6t)\hat{i} - (6t)\hat{j} - (t)\hat{k} + 7 \cdot \frac{-6i - 6j - k}{\sqrt{(-6)^2 + (-6)^2 + (1)^2}}$$

Consider the expression, $\frac{dr}{dt} = \vec{v}(t)$

$$= (-6t)\hat{i} - (6t)\hat{j} - (t)\hat{k} + 7 \cdot \frac{-6i - 6j - k}{\sqrt{(-6)^2 + (-6)^2 + (1)^2}}$$

$$= (-6t)\hat{i} - (6t)\hat{j} - (t)\hat{k} + \left(\frac{-46}{\sqrt{73}} \right)\hat{i} + \left(\frac{-46}{\sqrt{73}} \right)\hat{j} \left(\frac{-7}{\sqrt{73}} \right)\hat{k}$$

$$\frac{dr}{dt} = \left(\left(-6t - \frac{46}{\sqrt{73}} \right)\hat{i} + \left(-6t - \frac{46}{\sqrt{73}} \right)\hat{j} + \left(-t - \frac{7}{\sqrt{73}} \right)\hat{k} \right) dt \quad (2)$$

Integrating ②

$$\int dr = \int\left(-6t - \frac{46}{\sqrt{73}}\right)\hat{i}\,dt + \int\left(-6t - \frac{46}{\sqrt{73}}\right)\hat{j}\,dt + \int\left(-t - \frac{7}{\sqrt{73}}\right)\hat{k}\,dt$$

$$r(t) = \left(-\frac{6t^2}{2} - \frac{46t}{\sqrt{73}}\right)\hat{i} + \left(-\frac{6t^2}{2} - \frac{46t}{\sqrt{73}}\right)\hat{j} + \left(-\frac{t^2}{2} - \frac{7t}{\sqrt{73}}\right)\hat{k} + C_2 \quad ③$$

At $t=0$,

$r(0) = C_2$ ⓒ

Since the particle starts from the point $(8,8,8)$ at time $t=0$.

$r(0) = 8i + 8j + 8k$ ⓓ

Comparing ⓒ and ⓓ

$C_2 = 8i + 8j + 8k$

Substitute $C_2 = 8i + 8j + 8k$ in ③

$$r(t) = \left(-\frac{6t^2}{2} - \frac{46t}{\sqrt{73}}\right)\hat{i} + \left(-\frac{6t^2}{2} - \frac{46t}{\sqrt{73}}\right)\hat{j} + \left(-\frac{t^2}{2} - \frac{7t}{\sqrt{73}}\right)\hat{k} + (8i + 8j + 8k)$$

$$r(t) = -\frac{6t^2}{2} - \frac{46t}{\sqrt{73}}\hat{i} - \frac{6t^2}{2} - \frac{46t}{\sqrt{73}}\hat{j} - \frac{t^2}{2} - \frac{7t}{\sqrt{73}}\hat{k} + 8i + 8j + 8k$$

$$\boxed{r(t) = \left(-\frac{6t^2}{2} - \frac{46t}{\sqrt{73}} + 8\right)\hat{i} + \left(-\frac{6t^2}{2} - \frac{46t}{\sqrt{73}} + 8j\right)\hat{j} + \left(-\frac{t^2}{2} - \frac{7t}{\sqrt{73}} + 8k\right)\hat{k}}$$

A projectile is fired at a speed of 860 m/sec at an angle of 60°.
How long will it take to get 21 km downrange?

given that $V_0 = 860\ \frac{m}{sec}$ initial speed

$\theta = 60°$ angle of projection

$R = 21\ km$

$R = (V_0 \cos\theta)t \quad =$

divide 860 m/sec by 1000 to obtain 0.860 km/sec

$21 = (0.860 \cdot \cos 60)t$

$$t = \frac{21}{0.860 \cdot \cos 60}$$

$t \approx 48.84$ time it takes for the projectile to go 21 km downrange.

An athlete puts a 16-lb shot at an angle of 60° to the horizontal from 6 ft above the ground at an initial speed of 37 ft/sec. How long after launch and how far from the athlete does the shot land?

The shot remains in the air for 2.18 seconds.
(Round to two decimal places as needed.)

The shot travels 40.33 feet.
(Round to two decimal places as needed.)